Drain Output Resistance

I **fibbed**!

I have been saying that for a MOSFET in **saturation**, the drain current is **independent** of the drain-to-source voltage v_{DS}. *I.E.:*

$$i_D = K\left(v_{GS} - V_t\right)^2$$

In reality, this is only **approximately** true!

Due to a phenomenon known as **channel-length modulation**, we find that drain current i_D is **slightly dependent** on v_{DS}. We find that a **more accurate** expression for drain current for a MOSFET in **saturation** is:

$$i_D = K\left(v_{GS} - V_t\right)^2\left(1 + \lambda v_{DS}\right)$$

Where the value λ is a MOSFET **device parameter** with units of 1/V (i.e., V^{-1}). Typically, this value is small (thus the dependence on v_{DS} is slight), ranging from 0.005 to 0.02 V^{-1}.

Often, the channel-length modulation parameter λ is expressed as the **Early Voltage** V_A, which is simply the inverse value of λ:

$$V_A = \frac{1}{\lambda} \quad [V]$$

Thus, the drain current for a MOSFET in **saturation** can **likewise** be expressed as:

$$i_D = K\left(v_{GS} - V_t\right)^2 \left(1 + \frac{v_{DS}}{V_A}\right)$$

Now, let's **define** a value I_D, which is simply the drain current in saturation **if** no channel-length modulation actually occurred—in other words, the **ideal** value of the drain current:

$$I_D \doteq K\left(v_{GS} - V_t\right)^2$$

Thus, we can **alternatively** write the drain current in saturation as:

$$i_D = I_D \left(1 + \frac{v_{DS}}{V_A}\right)$$

This **explicitly** shows how the drain current behaves as a function of voltage v_{DS}. For example, consider a **typical** case case where v_{DS}=5.0 V and V_A= 50.0 V. We find that:

$$
\begin{aligned}
i_D &= I_D\left(1+\frac{v_{DS}}{V_A}\right)\\
&= I_D\left(1+\frac{5.0}{50.0}\right)\\
&= I_D\left(1+0.1\right)\\
&= 1.1\, I_D
\end{aligned}
$$

In other words, the drain current is **10% larger** than its "**ideal**" value I_D.

We can thus interpret the value v_{DS}/V_A as the **percent increase** in drain current i_D over its ideal (i.e., no channel-length modulation) saturation value $I_D = K\left(v_{GS}-V_t\right)^2$.

Thus, as v_{DS} increases, the drain current i_D will **increase slightly**.

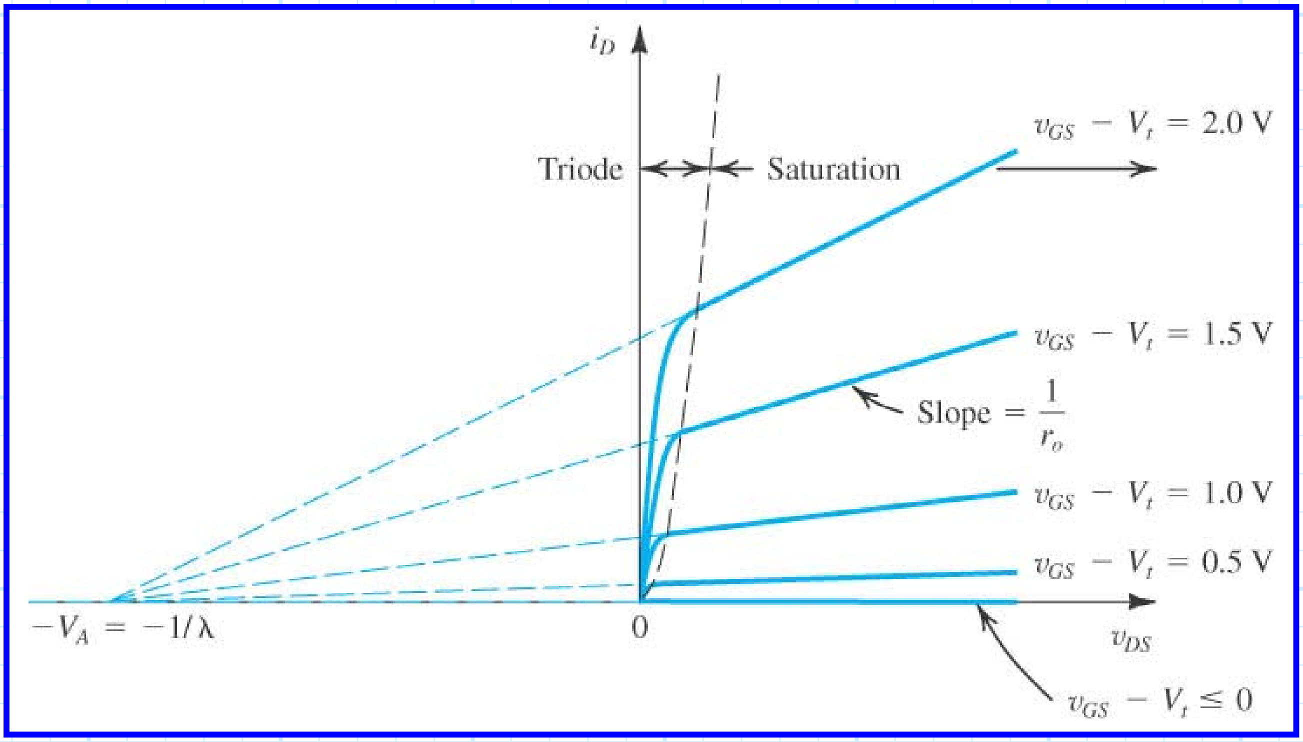
Now, let's introduce a **third** way (i.e. in addition to λ, V_A) to describe the "extra" current created by channel-length modulation. Define the **Drain Output Resistance** r_o:

$$
r_o \doteq \frac{V_A}{I_D} = \frac{1}{\lambda I_D}
$$

Using this definition, we can write the **saturation** drain current expression as:

$$
\begin{aligned}
i_D &= I_D\left(1+\frac{v_{DS}}{V_A}\right) \\
&= I_D + \frac{I_D}{V_A} v_{DS} \\
&= I_D + \frac{v_{DS}}{r_o} \\
&= K\left(v_{GS}-V_t\right)^2 + \frac{v_{DS}}{r_o}
\end{aligned}
$$

Thus, we **interpret** the "extra" drain current (due to channel-length modulation) as the current flowing through a **drain output resistor** r_o.



Finally, there are **three** important things to remember about channel-length modulation:

* The values λ and V_A **are** MOSFET **device parameters**, but drain output resistance r_o is **not** (r_o is dependent on I_D!).

* Often, we "**neglect** the effect of channel-length modulation", meaning that we use the **ideal** case for saturation--$i_D=K(v_{GS}-V_t)^2$. Effectively, we assume that $\lambda = 0$, meaning that $V_A = \infty$ and $r_o = \infty$ (i.e., **not** $V_A = 0$ and $r_o = 0$!).

* The drain output resistance r_o is **not** the same as channel resistance r_{DS}! The two are different in **many, many** ways:

$$i_D = K\left(v_{GS} - V_t\right)^2 + \frac{v_{DS}}{r_o}$$ for a MOSFET in **saturation**

$$i_D = \frac{v_{DS}}{r_{DS}}$$ for a MOSFET in **triode** and v_{DS} small

$$\therefore \quad r_o \neq r_{DS} \qquad !!!!!!!$$